Gap and screening in Raman scattering of a Bose condensed gas

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We propose different spectroscopic methods to explore the nature of the thermal excitations of a trapped Bose condensed gas: 1) a four photon process to probe the uniform region in the trap center: 2) a stimulated Raman process in order to analyze the influence of a momentum transfer in the resulting scattered atom momentum distribution. We apply these methods to address specifically the energy spectrum and the scattering amplitude of these excitations in a transition between two hyperfine levels of the gas atoms. In particular, we exemplify the potential offered by these proposed techniques by contrasting the spectrum expected, from the *non conserving* Bogoliubov approximation valid for weak depletion, to the spectrum of the finite temperature extensions like the *conserving* generalized random phase approximation (GRPA). Both predict the existence of the Bogoliubov collective excitations but the GRPA approximation distinguishes them from the single atom excitations with a gapped and parabolic dispersion relation and accounts for the dynamical screening of any external perturbation applied to the gas. We propose two feasible experiments, one concerns the observation of the gap associated to this second branch of excitations and the other deals with this screening effect.

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INTRODUCTION

Nature of the elementary excitations

The experimental discovery of the condensation of a Bose gas has confirmed the existence of the phonon-like nature of the collective excitations [1, 2]. The obtained measured energy spectrum not only is gapless as stated from the Hugenholtz-Pines theorem but is also in perfect agreement with the prediction of the Bogoliubov approach at zero temperature [3]. However, a second fundamental question arises as to whether these collective excitations are the elementary building constituents for the normal part of the fluid as assumed in the Bogoliubov approximation. Most standard textbooks rely on this quasiparticle hypothesis in order to determine the finite temperature gas properties [4, 5]. In contrast, in the theoretical description of a plasma, distinction is made between the elementary excitations (ions) and the collective ones (plasmons). As discussed in previous works [6–9], there are no fundamental reasons to exclude this distinction also in a Bose gas.

Precisely, suppose a bulk gas of total and condensed densities n and n_0 embedded in an volume V where atoms of mass m interact through the s wave channel with a scattering length a. The Bogoliubov approximation predicts that the elementary excitations of momentum $\mathbf{k}$ are phonon-like with a dispersion relation given by $\epsilon^B_{1,\mathbf{k}} = \sqrt{2gn_{\mathbf{0}}\epsilon_{\mathbf{k}} + \epsilon^2_{\mathbf{k}}}$ where $\epsilon_{\mathbf{k}} = \hbar^2\mathbf{k}^2/2m$ and $g = 4\pi a\hbar^2/m$. Nevertheless, its *non conserving* property (violation of the mass conservation law) [10] restricts its validity for a weakly depleted Bose gas and thus limits its use to low temperature. As opposed to that, the so-called generalized random phase approximation (GRPA) or equivalently the time-dependent Hartree-Fock (TDHF) approximation is instead *conserving* and valid for the whole range of temperature. This alternative approach distinguishes explicitly these collective phonon-like excitations from the atom-like elementary excitations with the parabolic dispersion relation $\epsilon^{HF}_{1,\mathbf{k}} = \epsilon_{\mathbf{k}} + g(2n - n_{\mathbf{0}}\delta_{\mathbf{k},\mathbf{0}})$ [6, 11–13]. The constant term corresponds to the Hartree and Fock (HF) mean field energy part and takes into account the absence of exchange interaction energy between condensed atoms. Therefore, an energy gap exists between the thermal and condensed atoms $\epsilon^{HF}_{1,\mathbf{k}} - \epsilon^{HF}_{1,\mathbf{0}} = gn_{\mathbf{0}} + \epsilon_{\mathbf{k}}$.

Superfluidity due to total screening

Another important reason to discriminate among the various theoretical approaches is to have an improved understanding of the superfluidity phenomenon. More precisely, we would like to answer the following question: Why, from a kinetic point of view, a superfluid can remain in a metastable motion without converting its kinetic energy into heat? Many explanations have been provided but, according to [4], *the situation is not entirely clear* as far as kinetic theory is concerned.

Instead, the equilibrium aspects based on the ensemble approach of the superfluid phenomenon of a Bose condensed gas are well understood. Using the η ensemble which breaks the $U(1)$ symmetry associated to the particle number conservation, one can describe the superfluid motion (condensed mode) relatively to the normal fluid (non condensed modes) [9]. Such a relative motion should not be considered as an equilibrium state but as a metastable state possibly subject to relaxation of a state of lower energy. Unfortunately, the ensemble approach

does not explain the physical reasons for such a metastability. It just tells that an artificial breaking of symmetry allows you such a description. Only a non-equilibrium treatment can provide these explanations and therefore confirm the validity of the assumptions used in the ensemble approach.

The kinetic theory so far developed in the Bogoliubov approximation allows for such a metastability in the weak depletion limit [14]. Particle exchange between the normal and superfluid are regulated through a balanced Beliaev process of transforming one collective excitations to two collective excitations. A complete different scenario appears in the GRPA as it accounts for the dynamical screening of any external time-dependent potential that perturbs the gas atoms [6, 8]. The ability of the macroscopic condensed wave function to deform locally its profile allows for a screening of any external perturbation that affects the energy transition probability of any atom-like excitation. In particular, under some stability conditions [8], a total screening forbids individual energy transitions involving a condensed atom. In this sense, the condensed atoms are *gregarious* since they respond only collectively to a perturbation via the creation of a phonon-like excitation. If the external potential originates from the presence of another thermal atom, this total screening prevents the binary collision between this thermal atom and any condensed one. Therefore, contrary to the Bogoliubov approach, the metastability of the relative motion between the normal and super fluids in GRPA is explained from the absence of this exchange collision process.

Nevertheless, atom exchanges between the normal and the super fluids should always exist in any kinetic description, in particular to guarantee the process of condensate formation. This is the case for the GRPA, but provided that instability conditions are satisfied [6]. For example, when the relative velocity between the two fluids exceeds the critical velocity given by the Landau criterion, the total screening phenomenon disappears and the binary collisions become again possible.

Experimental difficulties

Both gap and total screening phenomena have been predicted to appear in a Raman transition process between two hyperfine levels of a ^{87}Rb gas, but only in the bulk case [6]. In comparison to other methods like radio frequency (RF) or Bragg spectroscopy, the possibility of momentum transfer and the distinction between scattered and unscattered atoms enable these observations. However, an experimental realization is still not simple in the real case of a trap since the gas inhomogeneity, combined with the short duration of the applied coupling potential, leads to additional broadenings of the spectral lines that prevent the resolution of the gap and screening structure. In this context, a RF spectroscopy would have probed the whole gas which includes thermal atoms of the outer and inner condensate regions. Therefore, the distinction between various theoretical approaches is extremely difficult as long as the transition amplitude and the dispersion relation of thermal atoms have a strong spatial dependence.

Setup proposals

In this letter, we propose different methods to probe the atoms more efficiently than the RF spectroscopy: 1) the Raman scattering is a two-photon process that offers also the possibility to transfer the momentum $\mathbf{q}$ to the scattered atoms and observe their resulting momentum distribution after expanding the gas; 2) a four photon scattering process, where two sets of two beams cross in the trap center, addresses selectively the homogeneous region of the gas (see Fig.3).

We apply these methods for the case of a finite temperature trapped Bose gas in the GRPA, in a bid to challenge the Bogoliubov approach. To this end, we propose two concrete experimental setups that overcome the difficulties associated with the trap: 1) The gap is observed from the four-photon process; 2) The total screening is determined in a Raman scattering. Previous theoretical works [6, 8] argue in favor of the *conserving* GRPA. Nevertheless, a comparison with the *non conserving* Bogoliubov approximation is of relevance as long as the second branch of individual excitations has not been observed.

RAMAN SCATTERING

The GRPA approach

In a Raman transition, we start from atoms initially in the hyperfine level $|1\rangle = |F = 1, m_F = -1\rangle$. Each mode $\mathbf{k}$ is characterized by its initial population $N_{\mathbf{0}}$ and $N_{\mathbf{k}\neq 0} = 1/(\exp[\beta(\epsilon^{HF}_{1,\mathbf{k}} - \mu)] - 1)$ and its initial plane wave function $\psi_{1,\mathbf{k}} = \exp[i(\mathbf{k}.\mathbf{r} - \epsilon^{HF}_{1,\mathbf{k}} t)]/\sqrt{V}$ with the inverse temperature $\beta = 1/k_B T$ and the chemical potential $\mu = g(2n - n_{\mathbf{0}})$. The application of a perturbation coupling potential $V_{\mathbf{q}}(\mathbf{r}, t) = V_R \exp[i(\mathbf{q}.\mathbf{r} - \omega t)]$ at $t \geq 0$ transfers a small fraction of them into the second level $|2\rangle = |F = 2, m_F = 1\rangle$ of internal frequency ω_0. The determination of the second spinor component of the associated wavefunction $\psi_{2,\mathbf{k}}(\mathbf{r}, t)$ of the mode $\mathbf{k}$ evolves according to the time-dependant Hartree-Fock equation

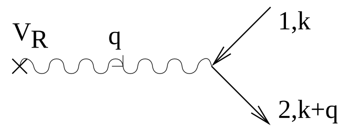

FIG. 1: Diagrammatic representation of the scattering of an atom by an external potential. An atom of momentum $\mathbf{k}$ is scattered into a state of momentum $\mathbf{k}+\mathbf{q}$ by means of an external interaction mediated by a virtual collective excitation of momentum $\mathbf{q}$.

[6]:

$$\left[i\hbar\partial_t + \frac{\hbar\nabla_{\mathbf{r}}^2}{2m} - \hbar\omega_0 - g_{12}\sum_{\mathbf{k}'\neq\mathbf{0}} N_{\mathbf{k}'}|\psi_{1,\mathbf{k}'}|^2\right]\psi_{2,\mathbf{k}} = \left[V_{\mathbf{q}} + g_{12}\sum_{\mathbf{k}'} N_{\mathbf{k}'}\psi^*_{1,\mathbf{k}'}\psi_{2,\mathbf{k}'}\right]\psi_{1,\mathbf{k}} \quad (1)$$

where we define the intercomponent coupling $g_{12} = 4\pi\hbar^2 a_{12}/m$. The solution is [6]:

$$\psi_{2,\mathbf{k}}(\mathbf{r},t) = \int_{-\infty}^{\infty} d\omega' \frac{\int_0^\infty dt' e^{i(\omega'+i0)(t'-t)} V_{\mathbf{q}}(\mathbf{r},t')\psi_{1,\mathbf{k}}(\mathbf{r},t)}{2\pi i\mathcal{K}_{12}(\mathbf{q},\omega')(\hbar\omega' + i0 - \hbar\omega_{\mathbf{k},\mathbf{q}})} \quad (2)$$

where $\hbar\omega_{\mathbf{k},\mathbf{q}} = \epsilon^{HF}_{2,\mathbf{k}+\mathbf{q}} - \epsilon^{HF}_{1,\mathbf{k}}$ and $\epsilon^{HF}_{2,\mathbf{k}+\mathbf{q}} = \hbar\omega_0 + \epsilon_{\mathbf{k}+\mathbf{q}} + g_{12}(n - n_{\mathbf{0}}\delta_{\mathbf{k},\mathbf{0}})$ is the atom mean field energy in the second level without the exchange term. These formulae resemble the one obtained from the non interacting Bose gas except for the HF mean field terms and the screening factor:

$$\mathcal{K}_{12}(\mathbf{q},\omega) = 1 - \frac{g_{12}}{V}\sum_{\mathbf{k}} \frac{N_{\mathbf{k}}}{\hbar\omega + i0 - \hbar\omega_{\mathbf{k},\mathbf{q}}} \quad (3)$$

Eq.(2) is interpreted in Fig.1 in terms of propagators whose poles determine the resonance frequencies. One pole is associated to the individual transition between atoms: $\omega = \omega_{\mathbf{k},\mathbf{q}}$ and the other is the zero of the screening factor and corresponds to the collective excitations associated to the gas rotation in the spin space: $\delta\omega = \omega - \omega_0 \sim [\epsilon_{\mathbf{q}} - (g - g_{12})n]/\hbar$ for $g_{12} \sim g$. Total screening corresponds to the singularity $\mathcal{K}_{12}(\mathbf{q},\omega_{\mathbf{0},\mathbf{q}}) \to \infty$ and prevents any single condensed atom scattering [6].

In a bulk gas, the transferred atom density for each mode is obtained from $n_{2,\mathbf{k}+\mathbf{q}}(t) = |\psi^{(1)}_{2,\mathbf{k}}(\mathbf{r},t)|^2 N_{\mathbf{k}}$ so that we deduce the total atom density[2, 6]:

$$n_2 = \sum_{\mathbf{k}} n_{2,\mathbf{k}} = \int_{-\infty}^{\infty} d\omega' \frac{4\sin^2(\omega' t/2)}{\hbar\pi\omega'^2}|V_R|^2 \chi''_{12}(\mathbf{q},\omega-\omega') \quad (4)$$

expressed in terms of the imaginary part of the intercomponent susceptibility function $\chi_{12}(\mathbf{q},\omega) = 1/(g_{12}\mathcal{K}_{12}(\mathbf{q},\omega))$.

The Bogoliubov approach

These results can be compared to the one obtained from the Bogoliubov *non conserving* approximation developed in [6, 7, 15] which is valid only for a weakly depleted condensate. This approach implicitly assumes that the elementary excitations are the collective ones forming a basis of quantum orthogonal states for the description of the normal fluid. Consequently, this formalism predicts no gap and no screening. The creation-annihilation operators $c^\dagger_{i,\mathbf{k}}(t), c_{i,\mathbf{k}}(t)$ describing the various components in the momentum space evolve according to $c_{1,\mathbf{k}}(t) = e^{-i\mu t}(\sqrt{N_{\mathbf{0}}}\delta_{\mathbf{k},0} + u_{+,\mathbf{k}}e^{-i\epsilon^B_{1,\mathbf{k}}t}b_{1,\mathbf{k}} + u_{-,\mathbf{k}}e^{i\epsilon^B_{1,\mathbf{k}}t}b^\dagger_{1,-\mathbf{k}})$ and $c_{2,\mathbf{k}}(t) = e^{-i(\mu+\epsilon^B_{2,\mathbf{k}})t}c_{2,\mathbf{k}}$. In this expression, besides the collective excitation modes of phonon of energy $\epsilon^B_{1,\mathbf{k}}$, a second collective mode of rotation appears with energy $\epsilon^B_{2,\mathbf{k}} = \epsilon_{\mathbf{k}} + (g_{12} - g)n_{\mathbf{0}}$. $\mu = gn_{\mathbf{0}}$ is the chemical potential, $b_{1,\mathbf{k}}$ is the annihilation operator associated to the quasi-particle such that $\langle b^\dagger_{1,\mathbf{k}}b_{1,\mathbf{k}}\rangle = 1/(\exp(\beta\epsilon^B_{1,\mathbf{k}}) - 1)$ and $u_{\pm,\mathbf{k}} = \pm((\epsilon_{\mathbf{k}} + gn_{\mathbf{0}})/2\epsilon^B_{1,\mathbf{k}} \pm 1/2)^{1/2}$. Reexpressing the intercomponent susceptibility

$$\chi_{12}(\mathbf{q},\omega) = \frac{iV}{\hbar}\int_0^\infty dt\ e^{i(\omega+i0)t}\langle[\rho^{12\dagger}_{\mathbf{q}}(0), \rho^{12}_{\mathbf{q}}(t)]\rangle \quad (5)$$

in terms of the autocorrelation function of the excitation operator $\rho^{\alpha\beta}_{\mathbf{q}}(t) = \sum_{\mathbf{k}} c^\dagger_{\alpha,\mathbf{k}}(t)c_{\beta,\mathbf{k}+\mathbf{q}}(t)/V$, we calculate in the Bogoliubov approximation:

$$\chi^B_{12}(\mathbf{q},\omega) = \sum_{\pm,\mathbf{k}} \frac{\delta_{\mathbf{k},\mathbf{0}}N_{\mathbf{0}}/2 \pm u^2_{\pm,\mathbf{k}}/(\exp(\pm\beta\epsilon^B_{1,\mathbf{k}}) - 1)}{V(\hbar\delta\omega + i0 \pm \epsilon^B_{1,\mathbf{k}} - \epsilon^B_{2,\mathbf{k}\pm\mathbf{q}})} \quad (6)$$

In contrast to the GRPA, Eq.(6) describes a spin rotation transition of the condensed fraction, one transition involves the excitation transfer from a phonon mode into a rotation mode and another the excitation creation in the two modes simultaneously.

Extension to the trap

These formulae can be easily extended to the case of a harmonic trap $V_H(\mathbf{r}) = \sum_i m\omega_i^2 r_i^2/2$ of frequency ω_i by considering the local density approximation (LDA) [5]. For a weakly inhomogeneous gas, the population in each mode becomes a local quantity $N_{\mathbf{k}} \to N_{\mathbf{k}}(\mathbf{r})$. By making this replacement, the thermal density $n_T(\mathbf{r}) = \sum_{\mathbf{k}\neq 0} N_{\mathbf{k}}(\mathbf{r})/V$, the energies $\epsilon^{HF}_{i,\mathbf{k}}(\mathbf{r})$, $\epsilon^B_{i,\mathbf{k}}(\mathbf{r})$, the screening factor $\mathcal{K}_{12}(\mathbf{r},\mathbf{q},\omega)$, the potential amplitude $V_R(\mathbf{r})$ and $n_{2,\mathbf{k}}(\mathbf{r},t)$ become local quantities as well. The zero mode density $n_{\mathbf{0}}(\mathbf{r}) = |\Psi_{\mathbf{0}}(\mathbf{r})|^2$ is determined from:

$$-\frac{\hbar^2\nabla^2_{\mathbf{r}}\Psi_{\mathbf{0}}(\mathbf{r})}{2m\Psi_{\mathbf{0}}(\mathbf{r})} + V_H(\mathbf{r}) + g(|\Psi_{\mathbf{0}}(\mathbf{r})|^2 + 2n_T(\mathbf{r})) = \mu \quad (7)$$

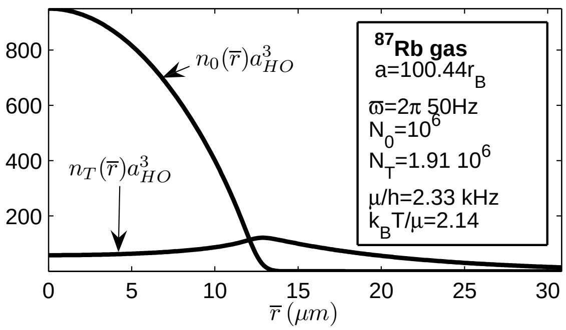

FIG. 2: Density profiles for the condensed and thermal clouds for typical values of gas parameters ($a_{H0} = \hbar/\sqrt{2m\overline{\omega}}$). $N_0 = \int d^3\mathbf{r}\, n_0(\mathbf{r})$ and $N_T = \int d^3\mathbf{r}\, n_T(\mathbf{r})$ are the total number of condensed and thermal atom respectively.

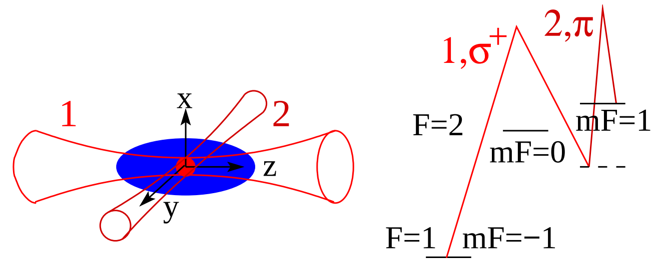

FIG. 3: Selective four lasers interaction with atoms within the trap center region. Two lasers σ^+ polarized along the z axis (1) drive the atoms through the intermediate states $m_F = 0$ and $m_F = 1$ while two others along the y axis (2) drive the atoms within the sublevel $m_F = 1$.

while the non zero ones are determined from the semi-classical expression:

$$N_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\exp\left[\beta(\epsilon_{1,\mathbf{k}}^{HF}(\mathbf{r}) + V_H(\mathbf{r}) - \mu)\right] - 1} \tag{8}$$

The set of Eqs.(7,8) is reduced to a one dimensional problem if we assume the ansatz $n_{\mathbf{0}}(\bar{r})$ where $\bar{r} = \sqrt{2mV(\mathbf{r})}/\overline{\omega}$ and $\overline{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$. This ansatz is exact for a spherical trap and is accurate in the Thomas-Fermi limit $\omega_i \ll gn(\mathbf{0})$. It leads to the profiles in Fig.2 for the condensed and normal fluids and shows excellent agreements with both experiments [16] and exact Monte-Carlo calculations [17] in the determination of the density profile of a trapped Bose condensed gas. These generalizations allow the determination of the transferred momentum distribution $N_{2,\mathbf{k}}(t) = \int d^3\,\mathbf{r}\, n_{2,\mathbf{k}}(\mathbf{r}, t)$ from which we deduce the transferred thermal atom number $N_{2,T}(t) = \sum_{\mathbf{k}\neq\mathbf{q}} N_{2,\mathbf{k}}(t)$.

EXPERIMENTAL PROPOSALS

The gap experiment: four-photon process

For $\mathbf{q} = 0$ and $g_{12} \sim g$, the Raman spectrum becomes discrete in a homogeneous gas. The resonance frequencies correspond to a gap $\hbar\omega_{\mathbf{k},\mathbf{q}}(\mathbf{r}) = -gn(\mathbf{r})$ associated to the exchange interaction energy for the single mode transition and to $(g_{12} - g)n(\mathbf{r})$ for the collective mode transition [11]. In comparison, if the condensed atom spectrum is quite similar, the thermal atom one displays differences in the Bogoliubov approximation. Since the energy difference $\epsilon_{2,\mathbf{k}}(\mathbf{r}) - \epsilon_{1,\mathbf{k}}^B(\mathbf{r})$ is $\mathbf{k}$ dependant, no gap is observed and the oscillations are smoothed out leading to a continuous spectrum.

In order to distinguish clearly between the discrete and the continuous spectra, the coupling potential acts specifically in the trap center in order to reduce inhomogeneous broadening. This is realized by means of four beams (see Fig.3) [18]: two gaussian astigmatic beams σ^+ polarized along the z axis of quantization with the intensity profile $I_1(\mathbf{r}) = I_{01}\exp(-2r_x^2/w_1^2(r_z) - 2r_y^2/w_2^2(r_z))$ and two others π polarized along the y axis with $I_2(\mathbf{r}) = I_{02}\exp(-2r_x^2/w_3^2(r_y) - 2r_z^2/w_4^2(r_y))$ where $w_i(s) = w_i(1 + (s\lambda)^2/(\pi^2 w_i^4))^{1/2}$. The sum of their frequency differences corresponds to the transition frequency ω. Provided that $\lambda \ll w_i$, we define an effective waist $\overline{w}$ such that:

$$\frac{1}{\overline{\omega}\,\overline{w}^2} = \frac{1}{\omega_x}\left(\frac{1}{w_1^2} + \frac{1}{w_3^2}\right) = \frac{1}{\omega_y w_2^2} = \frac{1}{\omega_z w_4^2} \tag{9}$$

In these conditions, the resulting potential $V_R(\mathbf{r}) = V_{R0}\exp(-2\bar{r}^2/\overline{w}^2)$ is optimized for an atom transfer in the most homogeneous region with $\mathbf{q} = 0$. To fix the idea, we choose $\lambda = 843nm$ and $\overline{w} = 7\mu m$ which reduces to about 10^4 the thermal atom effective number that can be specifically addressed. Transferring a small fraction of about 10% and for a detection resolution of about 100 atoms, we obtain a signal to noise ratio of about 10. A relative difference in the scattering lengths is also needed to observe the gap resonance and is obtained from the application of an external magnetic field [19]. These consideration leads to the spectra of Fig.4. Note the two orders of magnitude between the two peak intensities and the oscillatory behavior of period $1/t = 100Hz$ associated to the finite time resolution. The finite size of the beam provides an additional negligible frequency uncertainty of about $\hbar/(\sqrt{m\beta}\overline{w})$ in the resolution.

The screening experiment: Raman scattering

The absence of Raman transition due to screening is observed in the scattered atom momentum distribution. For a long time, the transient effects in Eq.(2) can be neglected leading to a constant transfer rate and, except

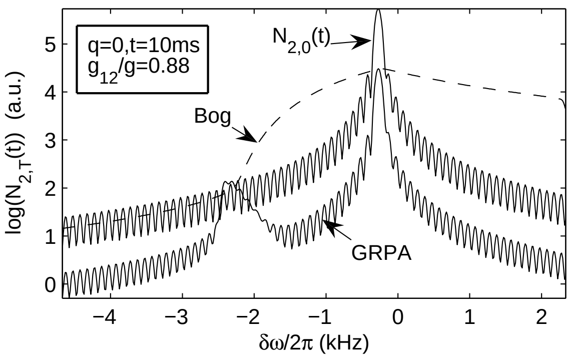

FIG. 4: Transferred thermal and condensed atoms vs. the detuning in the GRPA and Bogoliubov approximation.

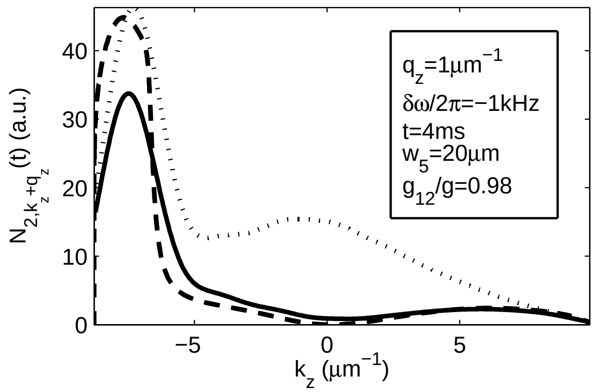

FIG. 5: Thermal atom distribution $N_{2,k_z} = \sum_{k_x,k_y} N_{2,\mathbf{k}}$ versus k_z in presence of screening using Eq.(10) (dashed line) and in absence (dotted line) and presence (full line) of screening taking into account finite interaction time corrections.

for the fact that the external potential is screened, we recover the Fermi golden rule:

$$\frac{N_{2,\mathbf{k+q}}(t)}{t} \overset{t\to\infty}{=} \int d^3\mathbf{r}\frac{2\pi V_R^2(\mathbf{r})n_{\mathbf{k}}(\mathbf{r})\delta(\omega-\omega_{\mathbf{k},\mathbf{q}}(\mathbf{r}))}{\hbar^2|\mathcal{K}_{12}(\mathbf{r},\mathbf{q},\omega)|^2} \quad (10)$$

Considering $g_{12} \sim g$, the transition energy is position dependant causing inhomogeneous broadening: $\hbar\delta\omega = k_z q_z/m + \epsilon_{\mathbf{q}} - gn(\mathbf{r})$. In the absence of screening, a resonance maximum appears for $k_z = 0$. The screening factor strongly reduces the Raman scattering and forbids it at this maximum i.e. $N_{2,k_x,k_y,q_z}(t)/t \overset{t\to\infty}{\to} 0$ thus avoiding the condensed atom transfer. For simplicity, let $\omega_x = \omega_y$. The atoms are transferred by means of a Raman transition resulting from two gaussian symmetric laser beams such that their wavevector difference $\mathbf{q}$ is along the z axis and their frequency difference is the transition frequency ω. For small q_z, the angle between the beams is small and the Raman potential has the gaussian circular profile $V_R = V_{R0}\exp(-2(r_x^2+r_y^2)/w_5^2(r_z))$. Once the atoms are transferred, the trap is switched off and after a time of flight, the density profile provides their momentum distribution.

A negative detuning is chosen in order to scatter the thermal atoms with k_z positive in the trap center region and negative otherwise. The graphs in Fig.5 illustrate well the total screening effect around $k_z = 0$ for which the macroscopic wave function deforms its shape in order to attenuate locally the Raman potential, thus preventing single atom scattering. The left part of the distribution ($k_z < -6\mu m^{-1}$) shows the thermal atoms coming from the outer condensate region. The choice of q_z is such that the LDA validity condition $q_z w_5 \gg 1$ is fulfilled but also such that, during the flight, the mean field energy does not affect much the momentum distribution. The interaction time must be much lower than the relaxation time associated with collisions $t \ll \tau \sim \sqrt{\beta m/8\pi a^2 n_T(\mathbf{0})}$ to avoid the equilibrium relaxation of the momentum distribution. Its finite value creates an energy uncertainty that alters the validity of Eq.(10) by not suppressing totally atom scattering at $k_z = 0$. Also, this time must be adequately chosen to suppress the condensed fraction due to the Rabi flopping associated to the collective mode: $N_{2,\mathbf{q}}(t) = 2\sin^2[(\hbar\delta\omega-\epsilon_{\mathbf{q}})t/2]N_{20,\mathbf{q}}$ where $N_{20,\mathbf{q}} \simeq 2\int d^3\mathbf{r}\, n_{\mathbf{0}}(\mathbf{r})[V_R(\mathbf{r})/(\hbar\delta\omega-\epsilon_{\mathbf{q}})]^2 = 0.19N_{2,T}$ for the case of Fig.5.

Phenomenological approach

Although theoretical statements argue in favor of GRPA, we cannot exclude that none of the two approximations reproduces correctly the physical observation. In such a case, we can use a phenomenological approach assuming a transition process from an excitation of unknown energy $\epsilon^X_{1,\mathbf{k}}$ to an excitation of energy $\epsilon^X_{2,\mathbf{k+q}}$ and a process of creation of two excitations of energy $\epsilon^X_{1,-\mathbf{k}}$ and $\epsilon^X_{2,\mathbf{k+q}}$. Using a four photon process interacting in the uniform region of the gas, the fraction of scattered atoms is then written under the form analog to Eq.(10):

$$\frac{N_{2,\mathbf{k+q_z}}}{t} \overset{t\to\infty}{\simeq} \sum_{\pm} A_\pm(\mathbf{q}_z,\mathbf{k})\delta(\omega \pm \epsilon^X_{1,\pm\mathbf{k}} + \epsilon^X_{2,\mathbf{k+q}}) \quad (11)$$

where $A_\pm(\mathbf{q}_z,\mathbf{k})$ represent the associated amplitude for such transition processes. Experimentally, the imaging in two dimensions allows only the determination of $F(k_x,k_z) = \int_\infty^\infty dk_y N_{2,\mathbf{k+q_z}}$. Thus the quantity (11) is determined from the Abel's transformation:

$$N_{2,\mathbf{k+q_z}} = -\frac{1}{\pi}\int_{\sqrt{k_x^2+k_y^2}}^\infty \frac{dF(y,k_z)}{dy}\frac{dy}{\sqrt{y^2-(k_x^2+k_y^2)}} \quad (12)$$

By varying the parameters q_z and ω, the resonance positions in the $\mathbf{k}$ space allow to reconstruct the dispersion relations $\epsilon^X_{1,\mathbf{k}}$ and $\epsilon^X_{2,\mathbf{k}}$ for the excitations.

CONCLUSIONS

We explored the many body properties of a trapped Bose gas that can be extracted from a two-level hyperfine transition in the GRPA and Bogoliubov approximation. The calculated spectra not only show the existence of a second branch of excitation but also the total screening of the external potential which prevents single condensed atom transitions. If the external potential originates from the presence of a thermal atom, this total screening prevents the binary collision between that thermal atom and any condensed one. In this scenario, the metastability of the relative motion between the normal and super fluids is explained by the absence of this exchange collision process [8]. The experimental observation of these phenomena will improve our understanding of the exact nature of the elementary excitations and of the origin of metastable motions in superfluids.

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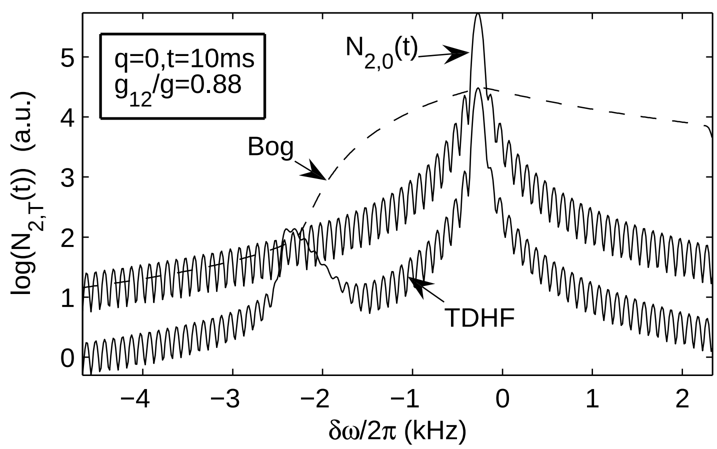
q=0,t=10ms
g12/g=0.88
N2,0(t)
Bog
TDHF
log(N2,T(t)) (a.u.)
δω/2π (kHz)
0
1
2
3
4
5
−4
−3
−2
−1
0
1
2